Reduced Order Modelling of Partial Differential Equations using Deep Neural Networks: Towards Optimality

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We develop a Deep Learning approach for the efficient approximation of the parameter-to-solution map in the context of parametrized PDEs. The research is motivated by the limitations of state-of-the-art algorithms, such as the Reduced Basis method, when addressing problems with a slow decay in the Kolmogorov n-width. We ground our construction on a nonlinear version of the Kolmogorov n-width, thanks to which we are able to purse an optimal model reduction, meaning that we scale down the problem to the minimal dimension granting the highest accuracy. After having established a solid theoretical background, we report numerical experiments where we compare our approach with POD-Galerkin methods. We run simulations in the presence of strong transport fields, singular terms and stochastic coefficients, which may be considered as prototype problems for forthcoming applications about fluid mechanics and electrodynamics (such as drift-diffusion equations) as well as porous media (e.g. subsurface flow and geomechanics).

References

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